Optimisation of maintenance policies for a system with multiple deteriorating components

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Condition-based maintenance (CbM) is a useful technique for scheduling maintenance policies aiming to reduce operating cost, improving the security of management, and ensuring the stable quality of the products. This paper models the deterioration process of a system composed of multiple components. Each deterioration process is modelled with the Wiener process. When a linear combination of the processes exceeds a pre-specified threshold, the age replacement policy will be carried out as the preventive maintenance for the system. Based on these two replacement policies, the optimized maintenance intervals are then sought. Besides, the paper also develops a cost process which considers the situation when the maintenance cost is higher than an expectation value, the decision-maker will prefer to replace the whole system but not repair it. Numerical examples are given to illustrate the optimisation process.

Keywords: Condition-based maintenance, Wiener process, Gamma distribution, Cost process

1. Introduction

Condition-based maintenance (CbM) is a useful technique for scheduling maintenance policies aiming to reduce operating cost, improving the security of management, and ensuring the stable quality of the products. In the CbM related literature, stochastic processes such as the gamma process Lawless and Crowder (2004); Cholette et al. (2019), the inverse Gaussian process Li et al. (2017); Hao et al. (2019), and the Wiener process Wen et al. (2018); Xie et al. (2019); Wang and Kang (2020) are widely used for different applications.

Basically, CbM is performed on the equipment once a parameter(s) related to the condition of the monitored system reaches a pre-specified value. Its purpose is to prevent the working efficiency of the system from reducing to an unacceptable condition or even if the system stops working completely, due to the ageing or deterioration of the system. It is therefore important to assess the status or remaining useful life of a system, which can further be used in deciding the future operation in order to maintain the system at a certain level of availability.

This paper intends to develop a deterioration process model, which is suitable for multi-component systems with multiple failure modes, and its corresponding maintenance policies. This can help the company to develop a long-term maintenance plan to reduce maintenance costs and increase the effectiveness of a system.

1.1. *Related work*

Caballé et al. (2015) propose a condition-based maintenance strategy by combining the non-homogeneous Poisson process (NHPP) and the gamma process(GP). It models a multiple deterioration processes within dependent degradation-threshold-shock model. They also point out that the dependence analysis between the causes of failure is a potential development and the variability of the threshold should be considered in future.

Zhu et al. (2015) simulate a wear process with a non-stationary Gamma process and the random shock damage with a generalized Pareto distribution satisfying Poisson arrivals. It is worthwhile to noticing that this study does not consider the impact of shocks or inspection costs which may influence the result of a long-term optimized maintenance policy.

Liu et al. (2017) propose a new CbM model based on three-state degradation and the influence of external environmental shocks. The deterioration process of the system is modelled by a two-state Wiener process with a Doubly Stochastic Poisson Process (DSPP). It considers two different

Proceedings of the 31st European Safety and Reliability Conference
Edited by Bruno Castanier, Marko Cepin, David Bigaud, and Christophe Berenguer

ISBN: 978-981-18-2016-8; doi:10.3850/978-981-18-2016-8_277-cd

thresholds, namely normal threshold and defective threshold which is depending on the system state.

Zhang et al. (2018) review some developments and applications of the Wiener process. It also summarize some challenges and problems which mainly include: the Wiener process with multiple time-scales, the Wiener process integrating various types of data, the Wiener process with state recoveries and the Wiener process with non-Markovian feature. Change points on degradation modelling and prognostics are largely occur randomly.

Wu and Castro (2020) investigate a CbM problem in which the deterioration process of a system is modelled by a weighted linear combination of multiple gamma processes for a pavement network. They also point that the degradation may follow different deterioration processes in one system. Besides, different failure modes can correspond to different thresholds which is a potential development as well.

Zhao et al. (2021) propose a multi-criteria mission abort policy which consider the normal and defective stages based on the time threshold. It also indicates that performance of the optimal policy is compared in detail against several heuristic policies. Besides, the dynamic risk for controlling policy is also a possible extension for phased mission systems.

Liu et al. (2021) propose a condition-based maintenance model in a finite-time horizon which consider a system with two heterogeneous dependent components with economic dependence. Moreover, this research points that the two-unit system in this paper can be extended to multi-unit systems by generalizing the degradation process and Bellman equation, and the maintenance level can be extended to imperfect repair in future.

1.2. *Novelty and contributions*

This paper models the deterioration process of a multi-component system, which is a linear combination of multiple Wiener processes. Specially, the paper investigates the cost process relating to the linear combination. Based on the cost process, it then formulates the expected cost of the life-cycle for the cases where the age replacement is applied.

The contribution of this paper includes

- development of a maintenance policy for a system whose deterioration process can be modelled by a linear combination of Wiener processes; and
- development of a cost process related to the linear combination of the deterioration processes.

1.3. *Overview*

The remainder of the paper is structured as follows. Section 2 lists notations and assumptions. Section 3 develops a linear combination of Wiener processes, derives the probability of the first time to exceed the pre-specified threshold, derives the cost process relating to the system deterioration process, and investigates the situation when the repair cost follows a probability distribution. Section 4 derives the maintenance policies base don the cost process. Section 5 shows some numerical examples. Section 6 concludes the paper, listed our findings, and proposes our future work.

2. Assumptions

2.1. *Notation*

Table 1 shows the notations used in this paper.

k	Types of failure modes.
$X_k(t)$	Degradation state of kth failure modes at time t.
$Y(t)$	Overall degradation of one system at time t.
μ_k	The drift of kth failure modes.
σ_k	The infinitesimal variance of kth failure modes.
a_k	The weight of failure mode k.
μ_Y	The drift of the overall degradation of one system.
σ_Y	The infinitesimal variance of the overall degradation of one system.
T_a	Interval time for the age replacement policy.
$C_i(T_a)$	Expected cost per unit time for the age replacement policy for maintenance policy i.
c_k	PM cost for every unit of kth failure modes.
c_m	Expected repair cost
c_r	Expected replacement cost
$C_k(t)$	Total cost of kth failure modes at t.
$U(t)$	Overall cost of one battery system at time t.
L	The threshold of the degradation level for a system.
L_c	The threshold of the cost for a system.

2.2. *Assumption*

- The system is new at time $t = 0$.
- Replacement is carried out every T time units.
- Degradations of different failure modes are Wiener processes, but have different parameters.
- There are n defects on the system, each defect develops from time $t = 0$. When a linear combination of the magnitudes of the defects exceeds a pre-specified value, the system needs replacement.

- Failure modes are independent from each others.

3. Model development

3.1. *Deterioration process*

We assume that the system has k deterioration processes, each of which follows a Wiener process.

Let $X_k(t)$ be the deterioration level of the kth deterioration process at time t. Then, $X_k(t)$ have the following assumptions:

- $X_k(0) = 0$, which also means that $W_k(0) = 0$;
- $W_k(t)$ has independent increments that follows the normal distribution. That is, for $0 < s < t$, $W_k(t - s) - W_k(s)$ follows $N(0, (t - s))$.
- $W_k(t)$ is continuous in t.

$X_k(t)$ has said having drift coefficient μ_k and variance parameter ${\sigma_k}^2$, the stochastic process of it is:

$$X_k(t) = \mu_k t + \sigma_k W_k(t), \quad (1)$$

where μ_k and σ_k are the parameters of failure mode k, respectively, $W_k(\cdot)$ is the standard Wiener process, which also can be called as the Brownian motion.

3.1.1. *Basic Properties*

The unconditional probability density function, which follows normal distribution with mean = 0 and variance = t, at a fixed time t:

$$f_{W_t}(x) = \frac{1}{\sqrt{2\pi t}} e^{-x^2/(2t)}.$$

We have $E[W_k(t)] = 0$ and $\text{Var}[W_k(t)] = t$.

These results follow immediately from the definition that increments have a normal distribution, centred at zero.

Thus, the expected value and the variance of $X_k(t)$ are given by: $E(X_k(t)) = \mu_k t$, and $V(X_k(t)) = {\sigma_k}^2 t$.

3.1.2. *A linear combination of Wiener processes*

Now let us assume $Y(t)$ is a linear combination of n Wiener processes. The overall degradation $Y(t)$ of the system is represented by

$$Y(t) = \sum_{k=1}^{n} a_k X_k(t), t \geqslant 0, a_k \geqslant 0, \quad (2)$$

where a_k is the weight of failure mode k. Fig. 1 shows the realisation of a linear combination of two Wiener processes.

Furthermore, the overall deterioration process $Y(t), t > 0$ is a stochastic process with the following properties (without the skew-normal random effects):

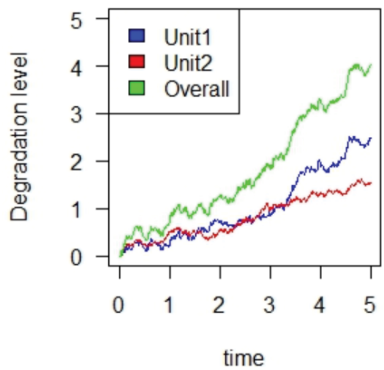

Fig. 1. Realisation of two deterioration processes and a linear combination

- $Y(0) = \sum_{k=1}^{n} a_k X_k(0) = 0$,
- $\Delta Y(t) = \sum_{k=1}^{n} a_k \Delta X_k(t)$ is an independent increment as well.

Thus, $Y(t)$ would be:

$$Y(t) = t \sum_{k=1}^{n} a_k \mu_k + \sum_{k=1}^{n} a_k \sigma_k W_k(t). \quad (3)$$

Let $\mu_Y = \sum_{k=1}^{n} a_k \mu_k$ and $\sigma_Y^2 = \sum_{k=1}^{n} a_k^2 \sigma_k^2$. Then $Y(t)$ follows the normal distribution $N(\mu_Y t, \sigma_Y^2 t)$.

3.1.3. *First time to exceed the pre-specified threshold L*

The distribution of the first hitting time of the process $\{Y(t), t \geq 0\}$, which starts from $Y(0) = 0$ should be obtained. The first hitting time $\omega_{Y(t)}$ is defined when $Y(t)$ reaches the degradation level L, according to the statistical characteristic of a Wiener process, $\omega_{Y(t)}$ should follow the inverse Gaussian distribution Ross et al. (1996); Pan et al. (2017), then

$$\omega_L = \inf\{t > 0 : Y(t) \geq L\}, \quad (4)$$

Then, the pdf of ω_L can be obtained as

$$\begin{aligned} f_{\omega_L}(t) &= \frac{L}{\sigma_Y \sqrt{2\pi t^3}} \exp\left(\frac{-(L - \mu_Y t)^2}{2{\sigma_Y}^2 t}\right) \\ &= \frac{L}{\sigma_Y \sqrt{\pi t^3}} \phi\left(\frac{-(L - \mu_Y t)}{\sigma_Y \sqrt{t}}\right), \quad (5) \end{aligned}$$

where $\phi(\cdot)$ denotes the standard normal cdf. Then, the cdf of ω_L is obtained by

$$\begin{aligned} F_{\omega_L}(t) &= P(Y(t) \geq L) \\ &= \Phi\left(\frac{-(L - \mu_Y t)}{\sigma_Y \sqrt{t}}\right) - \exp\left(\frac{2\mu_Y L}{{\sigma_Y}^2}\right) \quad (6) \end{aligned}$$

where $\Phi(\cdot)$ denotes the standard normal cdf.

3.2. *Repair cost process*

The repair costs of different failure modes are normally different. Therefore, c_k denotes cost of repairing the kth failure mode. Besides, we consider that the actual cost is dependent on the deterioration level of the failure model. It is worth noticing that, according to Wu and Castro (2020), the total cost $U(t)$, which is associated to $Y(t)$, is also a stochastic process and does not have a linear correlationship with $Y(t)$. As Y(t) is a Wiener process, U(t) is a Wiener process which is a sum of Y(t) with a drift.

Thus,

$$C_k(t) = a_k c_k X_k(t). \tag{7}$$

It is worth noticing that, when the cost of each inspection is considered, then the total repair cost $U(t)$ can be represented as

$$U(t) = \sum_{k=1}^{n} C_k(t) = \sum_{k=1}^{n} a_k c_k X_k(t), \tag{8}$$

$U(t)$ is a Wiener process with a linear drift.

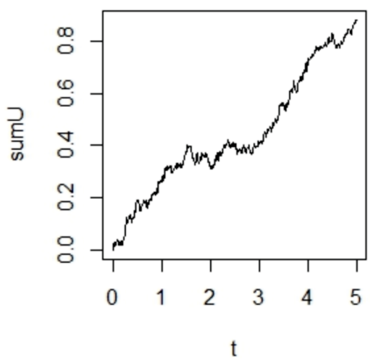

Fig. 2. Cost process of C(t)

As $X_k(t)$ follows the normal distribution with mean = $\mu_k t$ and variance = $\sigma_k^2 t$, the expected value and the variance of $C_k(t)$ are given by: $E(C_k(t)) = a_k c_k \mu_k t$ and $V(C_k(t)) = a_k c_k^2 \sigma_k^2 t$.

Then $U(t)$ has expected value and variance,

$$E(U(t)) = \sum_{k=1}^{n} a_k c_k \mu_k t = \mu_U, \tag{9}$$

and

$$V(U(t)) = \sum_{k=1}^{n} a_k^2 c_k^2 \sigma_k^2 t = \sigma_U^2, \tag{10}$$

respectively.

Obviously, both of $Y(t)$ and $U(t)$ have the same values μ_k and σ_k, respectively, so the covariance between $Y(t)$ and $U(t)$ is given by

$$\begin{aligned}\mathrm{Cov}(Y(t), U(t)) &= \mathrm{Cov}(\sum_{k=1}^{n} a_k X_k(t) \sum_{j=1}^{n} c_k X_j(t)) \\ &= \sum_{k=1}^{n}\sum_{j=1}^{n} a_k c_k \mathrm{Cov}(X_k(t), X_j(t)) \\ &= \sum_{k=1}^{n} a_k c_k \mu_k^2 t. \end{aligned}\tag{11}$$

The characteristic function of the bivariate normal distribution is given by

$$\begin{aligned}&\phi_{(Y(t),U(t))}(t_1, t_2) = \mathbb{E}[\exp(it_1 Y(t) + it_2 U(t))] \\ &= \mathbb{E}[\exp(it_1 \sum_{k=1}^{n} a_k X_k(t) + it_2 \sum_{k=1}^{n} a_k c_k X_k(t)] \\ &= \mathbb{E}[\exp(i \sum_{k=1}^{n} (a_k t_1 + a_k c_k t_2) X_k(t))] \\ &= \mathbb{E}[\exp(i \sum_{k=1}^{n} (a_k t_1 + a_k c_k t_2) X_k(t))] \\ &= \prod_{k=1}^{n} \mathbb{E}[\exp(i(a_k t_1 + a_k c_k t_2) X_k(t))] \\ &= \prod_{k=1}^{n} \phi_{X_k(t)}(a_k t_1 + a_k c_k t_2), \end{aligned}\tag{12}$$

then we can obtain

$$f_{Y(t),U(t)}(y, u) \tag{13}$$

$$\begin{aligned}&= \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_{(Y(t),U(t))}(t_1, t_2) e^{-it_1 y - it_2 u} \mathrm{d}t_1 \mathrm{d}t_2 \\ &= \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (\prod_{k=1}^{n} \phi_{X_k(t)}(a_k t_1 + c_k t_2))^{-it_1 y - it_2 u} \mathrm{d}t_1 \mathrm{d}t_2, \end{aligned}\tag{14}$$

then conditional probability $f_{U(t)|Y(t)(y,u)}$ is given by

$$f_{U(t)|Y(t)(y,u)} = \frac{f_{U(t),Y(t)(y,u)}}{f_{Y(t)}(y)}$$
$$= \frac{1}{4\pi^2 f_{Y(t)}(y)} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}$$
$$(\prod_{k=1}^{n} \phi_{X_k(t)}(a_k t_1 + c_k t_2))^{-it_1 y - it_2 u} \mathrm{d}t_1 \mathrm{d}t_2, \tag{15}$$

where

$$\phi_{X_k(t)}(a_k t_1 + c_k t_2)$$
$$= \exp\{\frac{\sigma_k[1-(1-2i\mu_k{}^2(a_k t_1 + a_k c_k t_2)\sigma_k{}^{-1})^{1/2}]}{\mu_k}\} \tag{16}$$

However, if we consider a real situation: after a period of time, $U(t)$ becomes so high that using a new piece of equipment to replace the old one may be a better choice. Also, the owner of the equipment may have an expectation overall cost: when $U(t)$ is larger than this expectation, they will buy a new piece of equipment. For example, we assume this expectation cost is L_U, which will be described in the next section. Similarly, we define

$$\omega_U = \inf\{t > 0 : U(t) \geq L_U\}, \tag{17}$$

Then, the pdf of ω_U can be obtained as

$$f_{\omega_U}(t) = \frac{L_U}{\sigma_U \sqrt{2\pi t^3}} \exp(\frac{-(L_U - \mu_U t)^2}{2\sigma_U{}^2 t})$$
$$= \frac{L_U}{\sigma_U \sqrt{\pi t^3}} \phi(\frac{-(L_U - \mu_U t)}{\sigma_U \sqrt{t}}). \tag{18}$$

Then, the cdf of ω_U is obtained by

$$F_{\omega_{L_U}}(t) = P(U(t) \geq L_U)$$
$$= \Phi(\frac{-(L_U - \mu_U t)}{\sigma_U \sqrt{t}}) - \exp(\frac{2\mu_U L_U}{\sigma_U{}^2}). \tag{19}$$

3.3. *Random effect*

Assuming that μ_k is influenced by other factors due to various reasons such as different operating environment and usage intensity,which are referred to as random effect. We assume that this random effect follows a gamma distribution over time. Then μ_k follows $\Gamma(\alpha_k, \beta_k)$ where α_k is the shape parameter and β_k is a scale parameter. Then μ_k has pdf as following:

$$f_{\mu_k}(x; \alpha_k, \beta_k) = \frac{1}{\beta_k^{\alpha_k}\Gamma(\alpha_k)} x^{\alpha_k - 1} \mathrm{e}^{-x/\beta_k} \tag{20}$$

where $\Gamma(\cdot)$ is a gamma function:

$$\Gamma(l) = \int_0^{\infty} y^{l-1} e^{-y} d\mathrm{t}$$

As $\mu_Y = \sum_{k=1}^{n} a_k \mu_k$ which have been described before, then the expected value and the variance of μ_Y are given by:$E(\mu_Y) = \sum_{k=1}^{n} a_k \beta_k \alpha_k$ and $Var(\mu_Y) = \sum_{k=1}^{n} a_k{}^2 \beta_k{}^2 \alpha_k$.

Then,

$$\mu_Y = \sum_{k=1}^{n} a_k \mu_k, t \geq 0, a_k \geq 0 \tag{21}$$

where μ_k follows the gamma distribution.

According to Moschopoulos (1985), the density function of μ_Y can be expressed by

$$z_{\mu_Y}(z) = B \sum_{k=0}^{\infty} \frac{\xi_k \beta_0^{-\tau-k}}{\Gamma(\tau + k)} z^{\tau+k-1} e^{-z/\beta_0} \tag{22}$$

where $\beta_0 = min a_k \beta_k$, and B and τ are

$$B = \prod_{k=1}^{n} (\frac{\beta_0}{a_k \beta_k})^{\alpha_k} \tag{23}$$

and

$$\tau = \sum_{k=1}^{n} \alpha_k \tag{24}$$

It is worth noticed that ξ_{k+1} can be obtained as

$$\xi_{k+1} = \frac{1}{k+1} \sum_{j=1}^{k} j \eta_j \xi_{k+1-j} \tag{25}$$

where $\xi_0 = 1$ and

$$\eta_k = \sum_{j=1}^{n} \alpha_j (1 - \frac{\beta_0}{a_k \beta_k})^k / k \tag{26}$$

Using μ'_Y to replace the original μ_Y, then, $Y(t)$ now can be replaced by $Y'(t)$, which can be presented by

$$Y'(t) = t\mu'_Y + \sigma_Y \sum_{k=1}^{n} W_k(t), \tag{27}$$

where μ'_Y follows a gamma distribution .

4. Maintenance policies

In this section, we consider the age replacement policy.

We have discussed the degradation process and the cost process based on the age replacement before. Then we attend to discuss the possible situations under these two processes:

- *Maintenance Policy A:* Under the degradation process, when the degradation level achieves the pre-specified threshold L, then maintenance activities will be taken. We assume that this event as A_1.
- *Maintenance Policy B:* Under the cost process, when the cost level achieves the pre-specified threshold L_U, then maintenance activities will be taken. We assume that this event as A_2.
- *Maintenance Policy C:* Only if both A_1 and A_2 have occurred, the age replacement will be conducted. Denote this event as $A_3 = A_1 \cap A_2$.
- *Maintenance Policy D:* If one of the two events, A_1 and A_2, occurs, the age replacement will be conducted. Denote this event as $A_4 = A_1 \cup A_2$.

Therefore, $G_1(t) := P(A_1) = F_{\omega_L}(t)$ and $G_2(t) := P(A_2) = F_{\omega_{L_U}}(t)$ and these functions can be obtained

$$\begin{aligned} G_3(t) &:= P(A_3) \\ &= P(A_1 \cap A_2) \\ &= P(A_1)P(A_2|A_1) \\ &= F_{\omega_L}(t)F_{\omega_{L_U}}(t|\omega_L) \\ &:= G_3(t), \end{aligned} \tag{28}$$

and

$$\begin{aligned} G_4(t) &:= P(A_4) \\ &= P(A_1 \cup A_2) \\ &= P(A_1) + P(A_2) - P(A_1 \cap A_2) \\ &= P(A_1) + P(A_2) - P(A_3), \end{aligned} \tag{29}$$

where symbol := is used to denote a definition.

4.1. *Age replacement policy*

For the age replacement policy, a preventive replacement is conducted after a continuous working time T_a when there is no failure occurs Barlow and Hunter (1960).

The expectation cost per time unit is given by

Let T_a be a replacement age, then the mean time between replacements $M(T_a)$ will be

$$\begin{aligned} M(T_a) &= \int_0^{T_a} tf(t)dt + t_0 P(X > T_a) \\ &= \int_0^{T_a} tf(t)dt + t_0(t - F(T_a)) \\ &= \int_0^{T_a} (1 - F(t))dt. \end{aligned} \tag{30}$$

Then, the mean cost per time unit is given by

$$C_i(T_a) = \frac{c_r + c_m G_i(T_a)}{\int_0^{T_a} (1 - G_i(t))dt}, \tag{31}$$

where $i = 1, 2, 3, 4$, corresponding to maintenance policies A, B, C, and D, respectively and T_a is the decision variable. Thus, the maintenance policy follows these principle.

- The inspection will be taken every T_a.
- Immediately after a preventive or corrective maintenance, the system rests its age to 0.
- Both c_r and c_m are constants.

By minimising $C_i(T_a)$, we can obtain the optimum T_a^* for the age replacement policy based on maintenance policies A, B, C, and D, respectively.

5. Simulation study

We use genetic algorithms to seek the optimal maintenance intervals.

We consider a system with two different failure modes. The deterioration process of the two defects is modelled with two Wiener process, each of which has different μ, α and β parameters. We assume that two modes has weights as following $a_1 = 0.3$ and $a_2 = 0.7$. α_1 and β_1 are 0.4 and 0.5 for the first failure mode, respectively. α_2 and β_2 are 0.3 and 0.6, respectively.

Thus, the linear combination of the two processes is given by

$$Y(t) = 0.3X_1 + 0.7X_2.$$

We assume that the system needs to be repaired when the degradation levels exceed the threshold $L_{w_L} = 5$ and the threshold $L_{w_{L_c}} = 3$, respectively. Replacement activities will be taken and the degradation level will be restored to zero when the component is completely replaced.

We assume that $c_r = 100$ and $c_m = 50$, then we can obtain the following result:

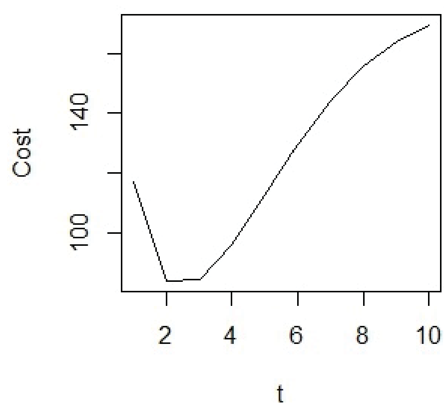

Fig. 3. Maintenance Policy A

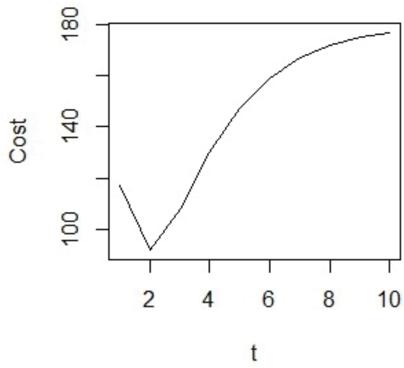

Fig. 4. Maintenance Policy B

Figures 3 and 4 show the expected cost per unit time under the maintenance policy A and B, respectively. Based on the genetic algorithms, we will obtain two optimized results as $T_{opt} = 2.443284$ (expected total cost is 82.12779) and $T_{opt} = 1.846838$ (expected total cost is 91.36122).

Besides, based on the age replacement policy, we can obtain the following results:

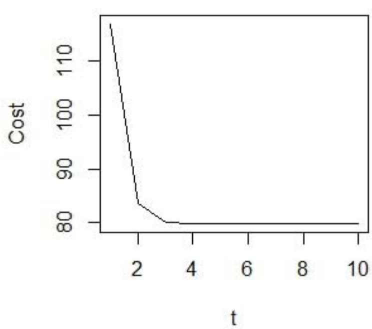

Fig. 5. Maintenance Policy C

Figures 5 and 6 show the expected cost unit for the age replacement policy under maintenance policy C and D, respectively.

- For maintenance policy C, the optimized point is $(T_{opt} = 8.06143)$ and the expected unit cost per time is 79.78846.

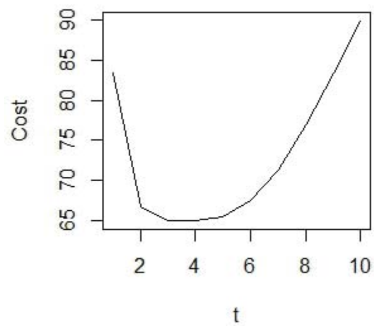

Fig. 6. Maintenance Policy D

- For maintenance policy C, the optimized point is $(T_{opt} = 3.552036)$ and the expected unit cost per time is 64.92507.

Then following table shows a summary result under different threshold:

Table 2 shows the simulation result in this table.

Maintenance	$L = 5, L_c = 3$	$L = 4, L_c = 2$
Policy A	T_{opt}=2.443284 (82.12779)	T_opt=3.014603 (80.19919)
Policy B	T_{opt}=1.846838 (91.36122)	T_opt=1.846838 (91.36122)
Policy C	T_{opt}=8.06143 (79.78846)	T_opt=7.03453 (80.56234)
Policy D	T_{opt}=3.552036 (64.92507)	T_opt=3.014603 (65.09959)

6. Conclusions

This paper discussed maintenance policies for a system with a linear combination of Wiener processes. When the degradation level of a linear combination of the processes exceeds a pre-specified threshold, the age replacement policy will be considered as the preventive maintenance for the system. Besides, we also develop a cost process which considers the situation that when the maintenance cost is higher than an expectation vale, the decision-maker will prefer to replace the whole system than repair it.

However, there are several limitations in our research:

6.1. *Limitations and possible development*

- The deterioration process of a system may be a non-linear combination of deterioration processes.
- Different component may follow different degradation models (e.g., one is the Wiener process and the others are gamma processes).
- The dependence among failure modes may exist and should therefore be considered.
- A multi-components system can be extended to a system with multi-phases degradation process can be considered. If so, changing points or turning points for a degradation process or a cost process should be considered.
- The arrival time of a failure mode may be considered so that the model can be used for some systems such as crack growth of pavements. Also, this can be extended to the arrival time of changing points for a degradation process or a cost process.

Our future work aims to investigate the above limitations.

References

Barlow, R. and L. Hunter (1960). Optimum preventive maintenance policies. *Operations research 8*(1), 90–100.

Caballé, N. C., I. Castro, C. Pérez, and J. M. Lanza-Gutiérrez (2015). A condition-based maintenance of a dependent degradation-threshold-shock model in a system with multiple degradation processes. *Reliability Engineering & System Safety 134*, 98–109.

Cholette, M. E., H. Yu, P. Borghesani, L. Ma, and G. Kent (2019). Degradation modeling and condition-based maintenance of boiler heat exchangers using gamma processes. *Reliability Engineering & System Safety 183*, 184–196.

Hao, S., J. Yang, and C. Berenguer (2019). Degradation analysis based on an extended inverse gaussian process model with skew-normal random effects and measurement errors. *Reliability Engineering & System Safety 189*, 261–270.

Lawless, J. and M. Crowder (2004). Covariates and random effects in a gamma process model with application to degradation and failure. *Lifetime data analysis 10*(3), 213–227.

Li, R., X. Liu, Y. Song, Z. Cai, and J. Li (2017). An optimal condition based maintenance strategy using inverse-gaussian degradation process. In *2017 Second International Conference on Reliability Systems Engineering (ICRSE)*, pp. 1–5. IEEE.

Liu, B., M. D. Pandey, X. Wang, and X. Zhao (2021). A finite-horizon condition-based maintenance policy for a two-unit system with dependent degradation processes. *European Journal of Operational Research*.

Liu, B., S. Wu, M. Xie, and W. Kuo (2017). A condition-based maintenance policy for degrading systems with age-and state-dependent operating cost. *European Journal of Operational Research 263*(3), 879–887.

Moschopoulos, P. G. (1985). The distribution of the sum of independent gamma random variables. *Annals of the Institute of Statistical Mathematics 37*(3), 541–544.

Pan, D., J.-B. Liu, F. Huang, J. Cao, and A. Alsaedi (2017). A wiener process model with truncated normal distribution for reliability analysis. *Applied Mathematical Modelling 50*, 333–346.

Ross, S. M., J. J. Kelly, R. J. Sullivan, W. J. Perry, D. Mercer, R. M. Davis, T. D. Washburn, E. V. Sager, J. B. Boyce, and V. L. Bristow (1996). *Stochastic processes*, Volume 2. Wiley New York.

Wang, H.-W. and R. Kang (2020). Modeling of degradation data via wiener stochastic process based on acceleration factor constant principle. *Applied Mathematical Modelling 84*, 19–35.

Wen, J., H. Gao, and J. Zhang (2018). Bearing remaining useful life prediction based on a nonlinear wiener process model. *Shock and Vibration 2018*.

Wu, S. and I. T. Castro (2020). Maintenance policy for a system with a weighted linear combination of degradation processes. *European Journal of Operational Research 280*(1), 124–133.

Xie, G., X. Li, X. Peng, F. Qian, and X. Hei (2019). Estimating the probability density function of remaining useful life for wiener degradation process with uncertain parameters. *International Journal of Control, Automation and Systems 17*(11), 2734–2745.

Zhang, Z., X. Si, C. Hu, and Y. Lei (2018). Degradation data analysis and remaining useful life estimation: A review on wiener-process-based methods. *European Journal of Operational Research 271*(3), 775–796.

Zhao, X., Y. Fan, Q. Qiu, and K. Chen (2021). Multi-criteria mission abort policy for systems subject to two-stage degradation process. *European Journal of Operational Research*.

Zhu, W., M. Fouladirad, and C. Bérenguer (2015). Condition-based maintenance policies for a combined wear and shock deterioration model with covariates. *Computers & Industrial Engineering 85*, 268–283.