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Fuzzy Map Comparisons for The Evaluation of Hydro-Morphodynamic Models

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Abstract

The validation of the accuracy of a numerical model counts to the numerous challenges in modeling hydro-morphodynamics. Many state-of-the-art approaches compare pixel-by-pixel topographic change using statistics such as Pearson's correlation coefficient. However, these statistics may not enable to differentiate between fairly well-performing models and complete non-sense models. This study employs spatially explicit map comparison techniques based on fuzzy sets to evaluate the efficiency of two-dimensional numerical hydro-morphodynamic models. We test a recently developed fuzzy numerical and a fuzzy kappa map comparison method for comparing observed topographic change with numerically modeled topographic change of a physical laboratory model. Fuzzy numerical map comparison introduces one level of fuzziness to compensate for spatial offset between modeled and observed values. Fuzzy kappa map comparison adds another level of fuzziness to account for similarities between categories. To this end, we categorize topographic change into categories of erosion and deposition. The results suggest that fuzzy kappa map comparison provides a suitable statement of the goodness of a numerical model, while fuzzy numerical map comparison shows limitations for expressing subjectively perceived correlation between modeled and observed topographic change. For instance, fuzzy numerical-based similarity coefficients are considerably high, even for a quasi-random non-representative model. We conclude that fuzzy map comparisons represent a powerful tool for the validation of hydro-morphodynamic numerical models.

Keywords: Fuzzy logic; Map comparison; Numerical modeling; Validation; Kappa statistics

1. INTRODUCTION

Modeling hydro-morphodynamic processes involves numerous challenges yet to be solved. One often-occurring question deals with the objective and realistic evaluation of the goodness (i.e., accuracy) of numerical models, where many state-of-the-art approaches build on pixel-by-pixel comparisons with statistics, such as the Pearson's correlation coefficient. These statistics merge data and model uncertainties into one global number that may hardly differentiate between non-sense and near-census reasonable model performance. For instance, many hydro-morphodynamic numerical models correctly predict landform patterns, but fail in exactly predicting topographic change at any pixel. Thus, pixel-by-pixel statistics often rate the model accuracy to be small, even for conceptually well-functioning models. For this reason, this study uses map comparison methods based on fuzzy sets theory to evaluate the efficiency of hydro-morphodynamic numerical models. In particular, we employ a fuzzy numerical (Hagen, 2006) and a fuzzy kappa (Hagen, 2003) method, which enable to compare modeled and observed topographic change maps while accounting for levels of fuzziness (vagueness of categories and in space) in the datasets. In a first step, we apply *fuzziness of location* to introduce a spatial tolerance in the comparison of modeled and observed topographic change maps with a fuzzy numerical method. Second, we add another level of fuzziness, namely *fuzziness of category*, with a fuzzy kappa method by categorizing erosion and deposition. Fuzziness of category considers that some categories are more similar to each other than others, and thus, it accounts for errors between the magnitudes of modeled and observed topographic change. Negreiros et al. (2021) introduced the theoretic background of both methods applied to hydro-morphodynamic models.

This study briefly recalls the algorithmic implementations of the fuzzy map comparison methods with novel features for evaluating hydro-morphodynamic numerical models. We then feature the application of the fuzzy map comparison methods to a new test case that compares laboratory (Schwindt et al., 2018) and numerically modeled (Reffo, 2017) topographic changes of a sediment trap. The results and discussion highlight the advantages of the tested methods and identify weaknesses of simplification hypothesis of the existing numerical model of the sediment trap.

2. METHODS

2.1 Fuzzy Map Comparison Methods

We use a fuzzy kappa method (Hagen, 2003) to investigate the goodness of fit of categorical topographic change data and a fuzzy numerical method (Hagen, 2006) to analyze the goodness of fit of continuous, numeric topographic change data. Both methods produce a measure of similarity that correlates modeled and observed topographic change maps (rasters) using fuzzy sets theory to account for errors in location. That is possible through *fuzziness of location*, which involves considering topographic change of a neighborhood of pixels. Thus, a pixel from the map of observations can be similar not only to its corresponding pixel in the modeled map, but also to its neighbors (Hagen, 2003). That means both methods use fuzziness of location to account for errors in location between the modeled and observed datasets. Figure 1a introduces the concept of fuzzy sets theory by exemplifying the attribution of a topographic change of 0.25 m to the categories of *deposition* and *little change*, which results in memberships of 0.5 in both categories. Figure 1b visualizes the concept of fuzziness of location. Fuzziness of location features a neighborhood of pixels and a membership function in the form of an exponential distance decay function ω. The neighborhood of pixels is a window of pixels within a finite pixel distance (integer) from the central pixel, which is termed *neighborhood radius*. The distance decay function expresses how much a neighboring pixel contributes to the representation of the fuzzy (central) pixel. Thus, the membership of each neighboring pixel depends on its distance to the central pixel d_{ij} and a constant *halving distance* d_{halv}, which is the distance (in pixels) from the central pixel at which the distance decay function (locational membership) equals 0.5.

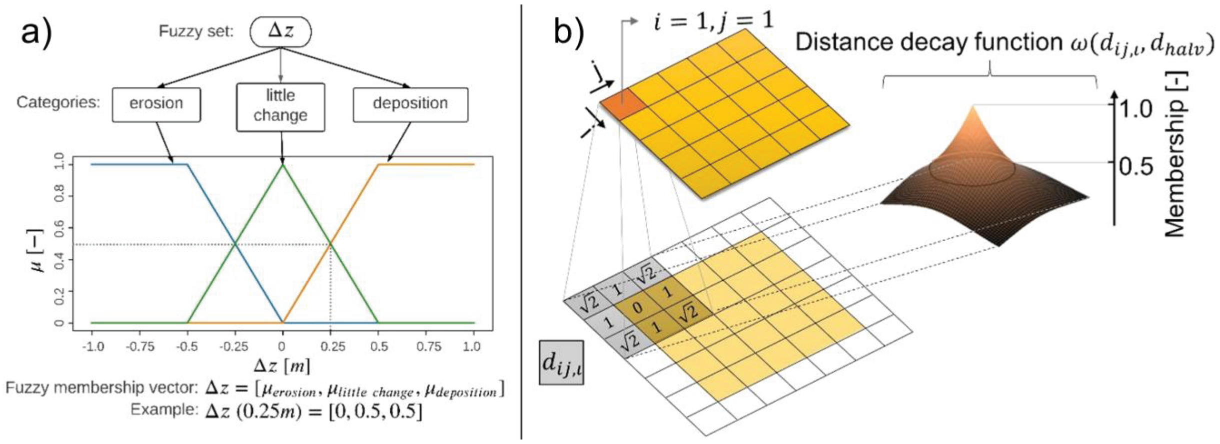

Figure 1. a) Qualitative example of a fuzzy set and b) the concepts of fuzziness of location with a neighborhood radius of one pixel (adapted from Negreiros et al., 2021). $\omega(d_{ij,\iota}, d_{halv})$ is the exponential distance decay membership function with a halving distance d_{halv} of one pixel; $d_{ij,\iota}$ is the distance between neighboring pixel at ι and central pixel at ij.

Per-pixel similarities (s_{ij}) are computed for both fuzzy numerical and fuzzy kappa method. We further refer to the fuzzy numerical and fuzzy kappa method with the superscripts fn and fk, respectively. In the fuzzy numerical method, topographic change of neighboring pixels in one map is compared with topographic change of the central pixel in another map and vice versa. The resulting similarities are weighted by the value of the distance decay function for every pixel. Thus, the calculation is executed in two directions: first, neighboring pixels in the modeled map are compared with central pixels in the observations map and second, neighboring pixels in the observations map are compared with central pixels in the modeled map. The maximum of the membership-weighted similarities in one direction constitutes a one-directional per-pixel similarity. A final, two-directional per-pixel similarity is calculated afterwards as the minimum between the two one-directional similarities. Ultimately, a global fuzzy similarity measure S^{fn} between the modeled and the observations map is calculated as the mean of the two-directional per-pixel similarities over the map pixels. The codes for the fuzzy numerical method, the pre-processing (rasterization), and the visualization of the results are made available to the public with an open-source license (Negreiros, 2020).

In addition to the fuzzy numerical method that is implemented in the novel codes, we also applied fuzziness of category with the fuzzy kappa method using the Map Comparison Kit (Visser & De Nijs, 2006). The fuzzy kappa method adds another dimension of fuzziness in addition to fuzziness of location through the categorization of data. For this reason, we bin topographic change into multiple categories of erosion and deposition. The natural breaks algorithm (Jenks, 1967) enables to classify topographic change into a selected number of categories by maximizing differences among categories while minimizing deviations within categories. Table 1 lists the intervals of the erosion and deposition categories computed with the natural breaks algorithm and as a function of minimum and maximum observed topographic change. The concept of fuzziness of category accounts for similarity between categories with membership values. To this end, we use a categorical membership of 0.5 between neighboring categories (e.g., between *little deposition* and *moderate deposition*) and 0.2 between categories with a distance of one category between each other (e.g., between *little deposition* and *much deposition*).

Table 1. Definition of categories for the sediment trap model in the lab using the natural breaks algorithm.

Category	Interval [m]
Little erosion	≤ - 0.024
Little change	] - 0.024, 0.024]
Little deposition	] 0.024, 0.064]
Moderate deposition	] 0.064, 0.097]
Much deposition	] 0.097, 0.140]

Similar to the fuzzy numerical method, the computation of per-pixel similarities with the fuzzy kappa method involves two one-directional comparisons considering both locational and categorical memberships (fuzziness of location and of category, respectively). A modified kappa coefficient (Cohen, 1960) calculates the overall map similarity while accounting for agreement that arises by chance between modeled and observed pixel values, which is called *expected agreement*. The modified kappa coefficient, notably the fuzzy kappa coefficient κ_f, is calculated with Eq. [1], where S^{fk} is the mean of per-pixel similarities (${s_{ij}}^{fk}$) calculated with the fuzzy kappa method and p_e is the expected agreement. Possible values for κ_f range from –∞ to 1.0, where κ_f = 1.0 indicates perfect agreement, κ_f = 0.0 indicates no agreement besides the expected agreement p_e, and κ_f < 0 indicates less agreement than the expected agreement p_e. Negreiros et al. (2021) explain the detailed calculation procedure for the fuzzy numerical and fuzzy kappa methods.

$$\kappa_f = \frac{S^{fk} - p_e}{1 - p_e} \tag{1}$$

Skill metrics have been used in other studies for assessing the performance of hydro-morphodynamic numerical models (Sutherland et al., 2004). To enable a comparison of the fuzzy set methods with skill metrics, we compute a quasi-random map of topographic change, which serves as a baseline to analyze the skill of a numerical model relative to a quasi-random, (physically) non-sense model. The quasi-randomness of the baseline map (raster) stems from the BitGenerator of the NumPy library (Harris et al., 2020), where the random values range from the minimum to the maximum observed topographic change. Finally, we calculate a recently developed skill score SS^{fn} for the fuzzy numerical method (Negreiros et al., 2021), which originates from the generic definition of skill score (Murphy, 1988) and resembles the fuzzy kappa coefficient (Eq. [1]):

$$SS^{fn} = \frac{S^{fn} - S^{fn}_{base}}{1 - S^{fn}_{base}} \tag{2}$$

The test approach builds on the comparison of numerically modeled and observed topographic change on a laboratory model of a sediment trap using the fuzzy numerical and the fuzzy kappa methods. In addition, we perform pixel-by-pixel map comparison between modeled and observed topographic change maps using Pearson's r. The merits of the fuzzy map comparisons and pixel-by-pixel map comparison are assessed by testing the methods on a quasi-random baseline map.

2.2 Testbed

Schwindt et al. (2018) used an experimental setup for developing and testing semi-permeable sediment traps at mountain rivers using a physical laboratory model. Sediment was supplied at the upstream end of the model by a system of conveyor belts and the discharge was controlled by a pump system. The sediment mixture consisted of fine and medium gravel (D_{84} = 13.7·10^{-3} m and D_{max} = 14.8·10^{-3} m), which was mixed with water in an adaptation reach upstream of the model's observation reach. In this study, we use data from a 3,300 s long experimental run featured in Schwindt et al. (2018) with a synthetic flood-like hydrograph and sediment supply (experiment number 06210). The sediment supply during the experimental run varied between 0.074 kg s^{-1} and 0.168 kg s^{-1} and the discharge varied between 5.5 m^3 s^{-1} and 12.5·10^{-3} m^3 s^{-1}. The sediment trap represented in the experimental setup consisted of a 1.20 m wide and 1.60 m long, quasi-octagonal (elongated) deposition area. A so-called guiding channel with a longitudinal slope of 5.5 % and a trapezoidal cross-section conveyed bankfull discharges up to $5.5 \cdot 10^{-3}$ m^3 s^{-1} (i.e. the minimum discharge during the experiment) across the deposition area. A rough bottom of the guiding channel and deposition area was fixed with cement grout (non-erodible bottom). A permeable hybrid barrier consisting of a bar screen and a slot check dam-like flow control structure was located at the downstream end of the guiding channel (and deposition area). The hybrid barrier was intended to trigger sediment deposition for flows exceeding the bankfull guiding channel discharge of $5.5 \cdot 10^{-3}$ m^3 s^{-1}.

Reffo (2017) simulated the physical model setup and the above-described experimental run with the two-dimensional numerical model Trent2D (Transport in Rapidly Evolutive, Natural Torrent; Armanini et al., 2009). Trent2D solves the depth-averaged Navier-Stokes equations with a finite volume scheme along a structured Cartesian grid (squared cells). Topographic change is modeled with empiric sediment transport equations, which Trent2D introduces into the Exner equation. However, the numerical model uses the following hypotheses that do not correspond to the conditions of the laboratory experiments: (1) the bottom of the guiding channel and sediment deposition area are mobile; (2) the sediment grain size distribution is uniform; (3) the hybrid barrier at the model outlet is substituted by an on-off flow condition and not geometrically implemented in the numerical grid.

In the laboratory, the topography of the sediment deposition area and guiding channel was measured before and after the experimental run with a motion-sensing camera (Kinect V2). The camera imagery was verified with additional laser measurements. Thus, the observed topographic change was calculated from the difference of the camera-derived topographic maps of the physical model setup before and after the experimental run. In this study, we use a coarsened resolution of the observed topographic change map with quadratic, 0.015 m long and wide pixels. The topographic change of the numerical model was extracted from the final deposition heights along the numerical grid.

2.3 Summary of the Test Approach

Our test approach applies the fuzzy numerical and the fuzzy kappa methods for comparing numerically modeled and observed topographic change during the above-described test run with a synthetic flood-like hydrograph and sediment supply. In addition, we perform a pixel-by-pixel map comparison of the modeled and observed topographic change maps using Pearson's r. We assess the results of the fuzzy and pixel-by-pixel map comparisons of the sediment trap by vetting them against the quasi-random baseline map.

3. RESULTS AND DISCUSSION

3.1 Comparison of Topographic Change

Figure 2 visually compares the observed topographic change after the experimental run in the lab (Figure 2a) with the numerically modeled topographic change (Figure 2b), and with the quasi-random baseline map (Figure 2c). Topographic change of a magnitude of up to 0.15 m can be observed along the guiding channel after the experimental run in the lab, while the modeled topographic changes show lower magnitudes of only up to 0.10 m. In addition, discrepancies between modeled and observed topographic change can be observed at the upstream boundary, where the numerical model simulates erosion (Δz = -0.05 m), but zero topographic change was observed in the physical experiment.

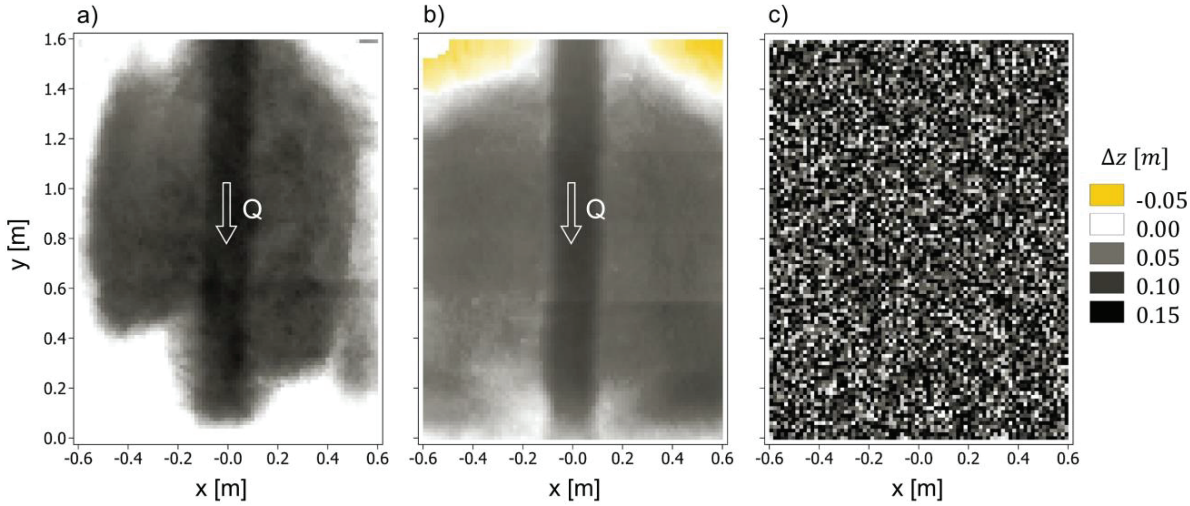

Figure 2. Topographic change maps of the sediment trap with a pixel size of 0.015 m (data source: Schwindt et al., 2018; Reffo, 2017): (a) observations map; (b) modeled map; and (c) quasi-random baseline map.

Figure 3 shows the comparison maps between modeled and observed topographic change resulting from the fuzzy numerical (Figure 3a) and fuzzy kappa methods (Figure 3b). The map comparisons use a neighborhood radius of 8 pixels (approximately 0.12 m) and a halving distance of 4 pixels (approximately 0.06 m). The fuzzy numerical map comparison indicates very low per-pixel fuzzy similarities (s_{ij}^{fn} = 0.00) between observed and numerically modeled topographic change at the upstream boundaries of the model (near the reservoir inlet; y > 1.4 m). In addition, very low per-pixel fuzzy similarities (s_{ij}^{fn} = 0) occur at the downstream boundaries of the model near the reservoir outlet, which is a result of overestimated deposition in the numerical model (cf. Figure 2). High per-pixel fuzzy similarities (s_{ij}^{fn} between 0.75 and 1.00) are apparent along the guiding channel.

The fuzzy kappa map comparison also reproduces the incorrect morphodynamic pattern produced with the numerical model at the downstream boundaries, where per-pixel fuzzy similarities take very low values (s_{ij}^{fk} close to zero). Yet, while the fuzzy numerical map comparison indicates low performance of the numerical model at the upstream boundary, the fuzzy kappa map comparison calculates moderate to high per-pixel fuzzy similarities (s_{ij}^{fk} from 0.50 to 1.00). The moderate to high per-pixel fuzzy similarities resulting from the fuzzy kappa method stem from the categorization of topographic change, which groups topographic changes between -0.024 and 0.024 m as “little change” (cf. Table 1). The fuzzy kappa method computes moderate per-pixel fuzzy similarities (s_{ij}^{fk} = 0.50) along the guiding channel. This observation suggests that the numerical model correctly simulates the deposition pattern along the guiding channel, but fails regarding the deposition magnitude.

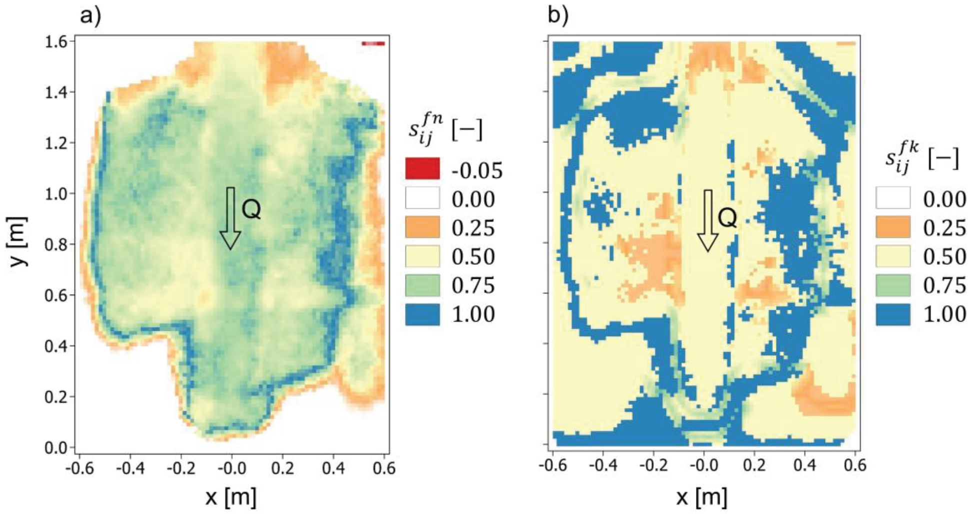

Figure 3. Comparison maps between modeled and observed topographic change: (a) fuzzy numerical and (b) fuzzy kappa map comparisons (pixel size = 0.015 m).

Table 2 summarizes the similarities resulting from the fuzzy numerical and fuzzy kappa methods. The fuzzy numerical method yields a moderate global fuzzy similarity (S^{fn}) between observed and modeled topographic change of 0.503. The global fuzzy similarity resulting from the comparison of the quasi-random baseline map and observed topographic change is only slightly lower with S_{base}^{fn}= 0.440. This observation suggests that the numerical model does not perform significantly better than the quasi-random baseline model. The overestimation of quasi-random baseline model performance by the fuzzy numerical method is known and has already been reported for other datasets (Negreiros et al., 2021). With a range of possible values between 0 (no skill) and 1 (maximum skill), the skill score SS^{fn} rates the performance of the numerical model poorly (SS^{fn} = 0.113) with respect to the baseline (reference) prediction.

The fuzzy kappa method yields lower similarities than the fuzzy numerical method (κ_f = 0.179; Table 2). Still, the κ_f value is significantly higher (factor 60) compared with the quasi-random baseline model (κ_{base} = 0.003). Thus, in contrast to the fuzzy numerical map comparison, the fuzzy kappa map comparison suggests that the numerical model performs significantly better than the quasi-random baseline model. Moreover, the values of κ_{base} are close to zero because of the expected agreement p_e that goes into the calculation of the fuzzy kappa similarity (Negreiros et al., 2021).

Table 2. Pixel-by-pixel statistics (Pearson's r) and fuzzy similarities of the numerical model and of the quasi-random baseline model.

Fuzzy numerical map comparison		**Fuzzy kappa map comparison**		**Pixel-by-pixel map comparison**	
S^{fn} [−]	S_{base}^{fn} [−]	κ_f [−]	κ_{base} [−]	r [−]	r_{base} [−]
0.503	0.440	0.179	0.003	0.044	0.013

A pixel-by-pixel map comparison of modeled and observed topographic change results in a small correlation coefficient (Pearson's r) of 0.044. In addition, a pixel-by-pixel map comparison between the quasi-random baseline map and the observations map results in a similar small correlation coefficient r_{base} = 0.013. Because the hydro-morphodynamic numerical model is driven by physical equations, the goodness-of-fit between modeled and observed topographic change could be expected to be significantly higher than the goodness-of-fit between quasi-random and observed topographic change. Yet, the pixel-by-pixel map comparison results in an almost-zero correlation (Pearson's r = 0.044), similar to the quasi-random baseline model (r_{base} = 0.013). Thus, Pearson's r rates the numerical model almost as poorly as a non-sense non-representative model. However, when comparing Figure 2a with Figure 2b, the human eye would suggest that those two maps resemble (i.e., correlate) much more than, for instance, Figure 2a with Figure 2c. In this sense, the fuzzy kappa method provides a more meaningful expression of the goodness of the numerical model than the fuzzy numerical global similarity or Pearson's r. In contrast, fuzzy numerical comparisons enable a better spatial evaluation of regions where the model performs particularly well or poorly. In addition, the skill score SS^{fn} (Eq. 2) rates the numerical model relative to the baseline reference model and this study confirms its relevance as discussed in Negreiros et al. (2021).

3.2 Why does the Numerical Model Perform so Badly?

The numerical simulation involves simplification hypotheses that may affect the goodness of results (see above). The following list attempts to rate the severity of model hypotheses by attributing them to physical processes:

1. The experimental setup had a fixed bed while Trent2D can only work with a mobile bed.
2. During the experiments, the sediment concentration was lower than the transport capacity. However, the numerical model assumes that the sediment transport corresponds to capacity conditions.
3. The numerical model does not accurately reproduce the continuous clogging of the check-dam like barrier. However, the clogging of the barrier drives the formation of a backwater zone in which sediment tends to deposit faster than under free-surface, normal flow conditions. Thus, in the physical experiment, sediment deposition might have started earlier, but sediment transport took place longer because the barrier was not clogged immediately from one time step to another.
4. The experiments were run with mixed grain sizes, but the numerical model could only simulate a single grain size.
5. The numerical model cannot account for infiltration processes and percolation processes in the porous space of deposited sediment.

Still, the numerical model correctly simulates sediment deposition patterns along the guiding channel (s_{ij}^{fn} and s_{ij}^{fk} between 0.50 and 1.00), though it fails to simulate the morphological inactivity observed at the downstream boundary (Figure 2a and 2b). The low fuzzy similarities (s_{ij}^{fn} and s_{ij}^{fk} = 0.00) of these regions (Figure 3a and 3b) reflect the overestimation of sediment deposition by the hydro-morphodynamic numerical model, which may be attributed to the above-listed set of hypotheses. In particular, modeling hypotheses 1, 2, 3, and 4 might have substantial implications for the numerical model output. For instance, low per-pixel fuzzy similarities at the upstream boundaries of the model stem from simulated negative topographic change, which is an artifact of the mobile bed (Hypothesis 1). In addition, the generally lower modeled topographic change in comparison with observations can be attributed to modeling hypothesis 2. The more uniform spatial distribution of the magnitude of modeled topographic change in comparison with observed topographic change can also be a result of the barrier implementation and the single grain size in the numerical model (Hypotheses 3 and 4).

4. CONCLUSIONS

The tested fuzzy map comparison methods represent a powerful tool for the validation of hydro-morphodynamic numerical models. The advantages of fuzzy map comparisons for evaluating numerically modeled topographic change are investigated in the form of a fuzzy numerical and a fuzzy kappa method. Both fuzzy map comparison methods provide an advantage over pixel-by-pixel statistics because they pardon spatial inaccuracy of the model. For instance, if the numerical model wrongly deposits a grain by a spatial offset, the fuzzy kappa method still appreciates that the model successfully modeled the deposition trend. In particular, the fuzzy kappa method shows a better appreciation of subjectively perceived correlation between modeled and observed topographic change. As an outlook, we suggest that fuzzy map comparisons can be applied also in other studies.

5. ACKNOWLEDGEMENTS

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