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The Case Study of the Alluvium Affecting Funchal City (Madeira Island, Portugal): Bivariate Modelling of Short-Duration Extreme Rainfall Events

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Abstract

Intense rainfalls and alluvium flooding are recurrent occurrences in Madeira Island. However, the understanding of how intense rainfalls relate to alluvium flooding is limited. The present research is intended to characterise the extreme rainfalls measured at the Funchal Observatório rain gauge and how they relate to the alluvium flooding in the region. While other studies on the subject applied univariate approaches to rainfall data, in this study the relationship between extreme events of rainfall and alluvium was addressed based on a copula approach as a way of describing the return periods of coupled extreme rainfalls with different durations. The AMS technique applied over a 34-year recording period was used to identify the extreme rainfall events and to assign to each of those events cumulative rainfalls in periods immediately before or/and after the same. By applying bivariate copulas to the coupled AMS-cumulative rainfall series, joint and conditional return periods were calculated and their relationship with alluvium flood events was analysed, including the deadly alluvium in late February 2010. The results obtained showed that the copula approach may contribute to a better understanding of the extreme rainfalls capable of triggering alluvium events. The coupled rainfall events that originated alluvium flood events tend to have higher return periods than those with no reports on such type of flood. It also proved the extreme exceptionality of the February 2010 event. This work also assists in understanding how intense rainfall events relate to alluvium flood events and the adequacy of copulas in hydrological studies.

Keywords: Funchal; Extreme Rainfall; Bivariate Copula; Alluvium Flooding; Return Period

1. INTRODUCTION

Intense rainfalls and alluvium flood events are common occurrences in Madeira Island (Portugal), some of them being deadly. However, there is a limited understanding of how intense rainfall events relate to alluvium flooding events. In this paper, the short duration extreme rainfalls measured at the Funchal Observatório gauge station are analysed aiming at understanding their relationship with the alluvium flooding reported by Sepúlveda (2011) as having occurred in the region (Gomes, 2021). For this purpose, bivariate statistical analysis was applied to "coupled" extreme rainfall events with different durations, thus allowing us to understand how the exceptionality of the rainfall events changes with their progression through time.

The main assumption of the study was that a similar extreme rainfall event may have different consequences in terms of alluvium flooding depending on the rainfall that preceded and/or followed it, that is, depending on the wetness condition of the watersheds. To explore such an assumption, bivariate series built upon extreme rainfalls coupled with cumulative rainfalls that occurred immediately before or/and after the extremes were analysed based on a copula approach aiming at identifying the association and dependence structure properties connecting those two random variables. By identifying the coupled extreme rainfalls that triggered alluvium events conclusions were made on how the exceptionality of the rainfall can be considered an indicator of the occurrence and consequences of alluvium flooding events. In this study special attention is given to the deadly late February 2010 event that resulted in more than forty causalities.

Whilst other studies on the subject take into consideration other hydrological factors and use a univariate approach applied to rainfall data, this study solely focuses on the bivariate behaviour of the extreme rainfalls with different durations as a trigger factor of alluvium flood events.

Some of the most important literature related to the subject of this paper was done by Lopes *et al.* (2020), Fragoso *et al.* (2012) and Levizzani *et al.* (2013). They studied various hydrological and climatological factors that could be related to or could have caused alluvium floods in Madeira with a focus on the February 2010 event (18-20.02.2010). A short description of this disastrous event can be found in Table 1 presented in Section 2.

To select the extreme rainfall events the Annual Maximum Series (AMS) technique was applied to hourly and daily rainfall series over periods of 34 and 80 years, respectively. Because the conclusions from the hourly analysis were equivalent to those from the daily analysis only the hourly one is addressed in this paper. The AMS technique extracts the maximum hourly rainfall of each hydrological year (that in Portugal runs from October 1 to September 30 of the following year) and compiles the values into one series, resulting in an hourly AMS with a length of 34 years for the Funchal Observatório case study. The use of the hydrological year is essential to ensure the randomness of the series since under the conditions prevailing in Madeira Island, but also in mainland Portugal, in each hydrological year, the hydrological conditions are re-initiated after the long dry summer season thus ensuring the temporal independence of some of the hydrological series, such as those used in the study. To understand the relationship between the exceptionality of the extreme rainfalls and the associated alluvium flood events, cumulative rainfalls with different durations were coupled to each annual maximum and characterized by conditional and joint return periods provided by a bivariate copula model. Among the several definitions of return period, Shiau and Shen (2001) describe it as the average elapsed time between occurrences of an event with a certain magnitude or greater than a threshold. The highest the return period the more exceptional the event is

The study area and the data used in the analysis are presented in Section 2. In Section 3 the modelling approach is described and in Section 4 some of its main results are presented. Section 5 includes a further discussion of the results and the main conclusions from the analysis.

2. CASE STUDY AND DATA

Madeira is a volcanic island located in the North Atlantic Ocean with an area of 741 km^2, a length of 57 km and a maximum width of 22 km. Centred at 32◦ 44.34′ N and 16◦ 57.91′ W, approximately 600 km northwest of the Western African coast (Figure 1), Madeira has a steep topography consisting of an enormous central E-W oriented mountainous system which divides the island mainly into north and south from an orographic perspective. According to Koppen's classification, the climate is predominantly temperate with dry and warm to hot summers approaching the coastal zones of Madeira.

The rainfall occurs predominantly in the north-facing slope as a result of the topography combined with the prevailing N-E trade winds. The rainfall is concentrated in the period from October to mid-April, while in summer (from June to August) is very low. The highest average annual values, exceeding 2200 mm, are observed in the northern slope and especially in the central highland region of the island while the smallest rainfalls, less than 650 mm, occur in the lowland areas of the southern slope, including in the Funchal area, the capital of Madeira and where the rain gauge adopted in the study is located.

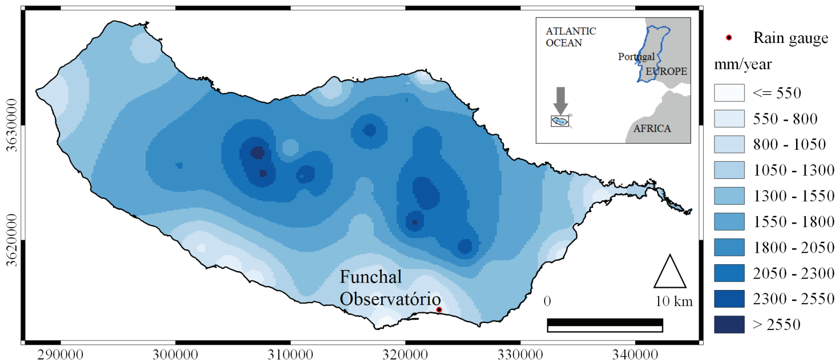

Figure 1. Madeira Island and Funchal Observatório rain gauge location over a map of the mean annual rainfall.

The two data sets required by the developed approach comprehended rainfall data and alluvium data. The first set included the hourly rainfall records at Funchal Observatório rain gage from the 1st of October 1980 to the 30th of September 2014 provided by the IPMA – Instituto Português do Mar e da Atmosfera, I. P. (34 hydrological years). The second set of data, related to recorded alluviums from 1601 to 2010, was collected from Sepúlveda (2011). For each alluvium event, this author provides some information on the weather conditions, location of the occurrence and damages.

Aiming at identifying possible cause-effect relationships between annual maximum hourly rainfalls and alluviums, three types of criteria were developed and applied to the alluviums events systematised by Sepúlveda (2011): the **temporal**, the **spatial**, and the **substantive** criteria. However, it is important to clarify that all the annual maximum hourly rainfalls were used to set up the copula models and not only those that were further related to alluviums. In other words, the analysis based on copulas considered all the extreme rainfall events independently of if they could be considered of having triggered alluviums or not.

The temporal criterion considers that there might exist a cause-effect relationship between an extreme rainfall and an alluvium flood event if the annual maximum rainfall occurred within the previous 6 days of the identified alluvium. The spatial criterion considers that the alluvium must have been said to occur specifically in Funchal area or to have impacted the southern slope of the island (where Funchal area is located) or the whole island. Finally, the substantive criterion was defined as having caused either floods or landslides or damaging impacts to civil infrastructure and human life.

For the period with hourly rainfall data, the eleven coupled rainfall-alluvium events that simultaneously met the three previous criteria are systematized in Table 1. The table specifies the hydrological year of occurrence of each alluvium, its date, well as the date of the annual maximum rainfall that was assigned to the alluvium and, finally, a brief description of the alluvium event.

Table 1. Association between annual maximum hourly rainfalls and alluviums. Hydrological year and dates of occurrence and description of the alluviums (reproduced from Gomes, 2021).

Hydrological year	AMS date	Alluvium date	Alluvium description
1989/1990	18/09/1990	18/09/1990	"Falling of blocks in Curral das Freiras: "landslide happened after the strong rainfall which happened between 14h and 15h" (DNM, 1990). Floods also took place in Funchal"
1991/1992	29/10/1991	29/10/1991	"In Funchal the rain caused floods and damage to the sewege systems. Also, in Câmera de Lobos floods were registered in the residences and anomalies in the sewer systems."
1993/1994	29/10/1993	29/10/1993	"Funchal was woken up starteled. The intensive rain and streams filled with rubble caused a catastrophe. (…) The trajedy struck various points of the island"
1994/1995	07/10/1994	07/10/1994	"Great rainfall registered during all of the day and provoked some floods a landlides in diverse areas of the island."
1995/1996	22/03/1996	22,23/03/1996	Strong storm with great discharge of water in all the island. Landslides, falling of trees and the obstruction of roads happened.
1997/1998	01/02/1998	07/02/1998	"A bit everywhere, with land giving waybecause of the wight of the rainfall water" (DNM, 8 Fev. 1998).
1999/2000	10/10/1999	10/10/1999	"Strong rainfall in Funchal, followed by landslides."
2001/2002	18/11/2001	18,19/11/2001	"Storm mainly on the south side of the island provoked floods, landslides and the falling of trees."
2002/2003	24/11/2002	24/11/2002	"Storm over all the island, mainly in the south and west, provoked landslides, floods and obstructions of roads."
2006/2007	07/04/2007	7,8,10,11/04/2007	"Intensive rainfall provoked floods in Funchal." "Intensive rain provoked loss of stones in access roads to Curral das Freiras, and also floods."
2009/2010	20/02/2010	18-20/02/2010	"All of the south side of the island was affected by the by the storm. The final official balance indicates that 43 people died, 8 remain lost, 120 were injured and 800 habitations suffered damages, 400 of which there was a total loss or are needing a deep intervention, with a loss of 36 million Euros. (...) The Comissaão Partiaária Mista defined the value of loss at 1080 million Euros,"

3. MODELLING APPROACH

If only annual maximum hourly rainfalls were analysed in a univariate perspective, under the hypothesis that the alluviums could be associated with such extreme rainfalls, the in-time internal relationship of said rainfall events could not be addressed. In the bivariate analysis, a second variable related to cumulative rainfall was considered. This means the simple usage of an AMS approach may be insufficient for a deeper understanding of the exceptionality of a rainfall event because it only portraits a very small time window (1 h in the case of hourly rainfall AMS) and not the rainfall conditions during its occurrence. Since alluviums are related to rainfall that occurs along time and its effects on the humidity content and cohesion of the ground soils, a more meticulous understanding of the rainfall event before the alluvium and on its change in time is necessary. Under this understanding and assuming that there would be some correlation between annual maximum rainfalls and cumulative rainfalls temporally contiguous to those maxima, each annual maximum rainfall was coupled with the cumulative rainfall that occurred before or/and after it. For this purpose, different accumulation periods were considered, as further discussed. Such a bivariate model must also have the ability to enjoin the two variables into one distribution so that in the qualitative hydrological sense the two variables can be looked at as one coupled extreme rainfall occurrence, eventually assigned to an alluvium flood event. For this purpose, the bivariate copula model was used.

Such a model is in essence a bivariate distribution from which joint or conditional probabilities can be calculated, and it allows for an understanding of a possible non-linear relationship between two variables. In its application to Funchal Observatório rain gauge records, the hourly AMS values were used as the variable number 1, X_0, i.e., the defining characteristic of the extreme rainfall event, while the cumulative rainfalls before and/or after each annual maximum were set as variable number 2 of the bivariate analysis, X_n^B, X_n^A and X_n^{BA}, where "n" represents the number of hours considered when computing the cumulative rainfall and X_n^B, represents the cumulative rainfall "n" hours before each annual maximum, including this maximum, X_n^A the same for "n" hours after each annual maximum, and X_n^{BA} a mix of the two previous scenarios, i.e., cumulative rainfall in "n" hours surrounding (before and after) each annual maximum, including the maximum. Figure 2 helps to understand how the cumulative rainfall series were obtained. As represented in the figure, the accumulation periods may include sub-periods (hours) without rainfall. The series of X_0, X_n^B, X_n^A and X_n^{BA}, with "n" from 1 to 6, used in the study are presented in Table 2, included in the next page.

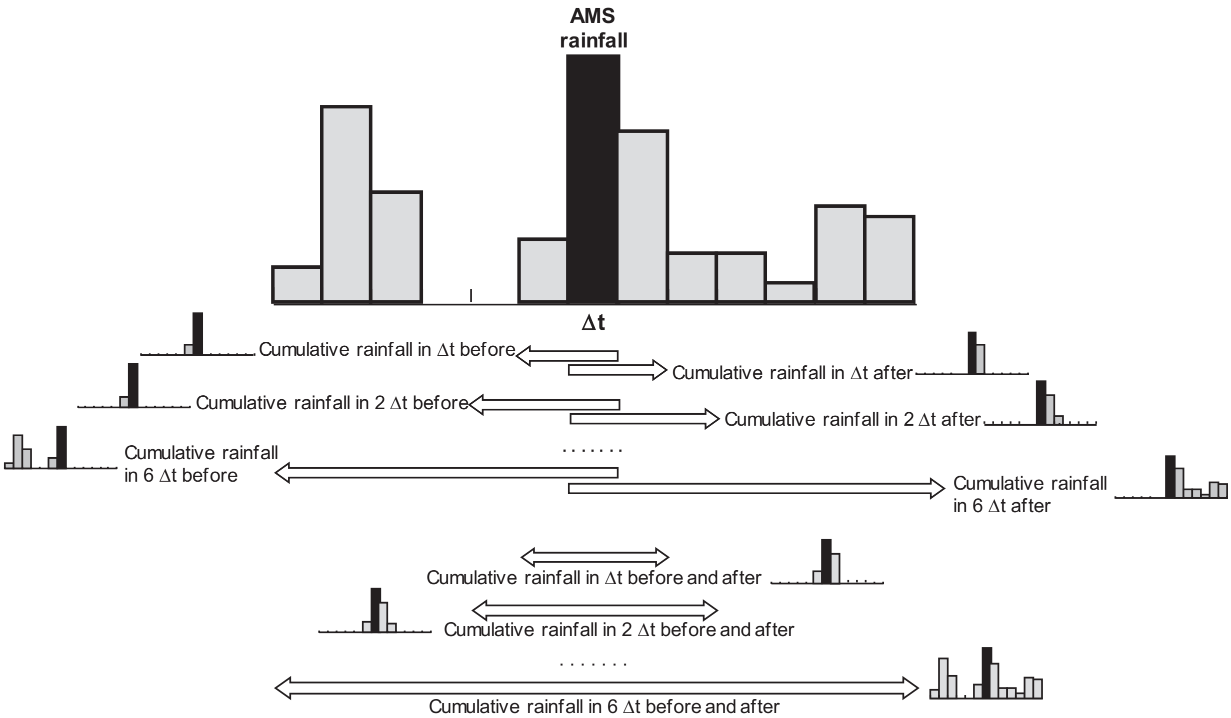

Figure 2. Schematic representation of the procedure of creating coupled AMS rainfalls and cumulative rainfalls in contiguous time steps with a duration of Δt=1 h (reproduced from Gomes, 2021).

For each of the considered scenarios – cumulative rainfalls before, X_n^B, after, X_n^A, and surrounding the AMS hourly rainfalls, X_n^{BA}, represented simply by X_n – each coupled (X_0, X_n) series provides the information required to set up the copula models. In each scenario, six different copulas (each one for a given duration of the cumulative rainfall, between 1 and 6 h) were pre-selected for analysis, obviously assuming that the variables being associated should not be independent but on the contrary should possess some correlation, as previously said. To estimate the correlation between any two variables used in the copulas, Kendall's rank correlation coefficient, τ, was applied (Kendall, 1938).

With the (X_0, X_n) series established, the next step of the approach required the identification of the statistical distributions and their parameters that best describe the marginal distributions of each two paired variables. The statistical distributions considered were the Normal, the Gamma, the Weibull, the Exponential, the Cauchy, the Logistic, and the Lognormal. The parameter estimation methods applied were the Maximum Likelihood Estimation method, the Moment Matching Estimation method, the Quantile Matching Method with quantiles set at 0.25 and 0.75, and, finally, the Maximum Goodness-of-Fit Estimation method with Cramer-Von Mises, Kolmogorov-Smirnov and Anderson-Darling distances. For each of the previous marginal distributions and parameters estimates the relative fitting quality was assessed based on the Log-Likelihood Function (LLF), the Akaike Information Criterion (AIC), and the Bayesian Information Criterion (BIC). By ranking the results from these tests, the best marginal distributions were selected to set up the copula models. The three quality criteria all agreed in producing the same ranking for the marginal distribution fitting. The term "relative fitting quality" refers to when a distribution is better fitted to a series than another distribution that was also tested. But, it does not demonstrate that the distribution is sufficiently well fitted in an absolute perspective, being one of the causes of the so-called epistemic uncertainty in hydrology-related studies.

Table 2. Hourly AMS series, X_0, and coupled cumulative rainfall series, X_n^B, X_n^A and X_n^{BA} (mm), from 1 to 6 h, at Funchal Observatório rain gauge.

Date of X_0	Hour of X_0	X_0	X^B_1	X^B_2	X^B_3	X^B_4	X^B_5	X^B_6	X^A_1	X^A_2	X^A_3	X^A_4	X^A_5	X^A_6	X^{BA}_1	X^{BA}_2	X^{BA}_3	X^{BA}_4	X^{BA}_5	X^{BA}_6
11/11/1980	08:00	11.4	13.1	13.1	13.4	13.4	13.4	13.4	19.0	19.0	19.2	19.2	20.0	20.0	20.7	20.7	21.2	21.2	22.0	22.0
21/11/1981	12:00	16.2	22.3	24.4	25.5	25.5	25.5	25.5	20.5	22.1	22.2	22.2	22.2	22.2	26.6	30.3	31.5	31.5	31.5	31.5
23/09/1983	20:00	10.9	14.6	14.6	14.6	14.6	14.6	14.6	11.6	11.7	11.7	11.7	11.7	11.7	15.3	15.4	15.4	15.4	15.4	15.4
21/09/1984	23:00	22.0	22.0	22.0	22.0	22.0	22.0	22.0	28.5	31.7	34.7	34.7	35.5	46.7	28.5	31.7	34.7	34.7	35.5	46.7
06/01/1985	08:00	24.7	38.3	42.8	43.3	43.3	43.3	43.3	36.9	37.0	37.0	37.2	37.2	37.2	50.5	55.1	55.6	55.8	55.8	55.8
23/10/1985	01:00	28.6	33.5	33.5	33.5	33.5	33.5	33.5	32.6	32.6	32.6	32.6	34.2	37.6	37.5	37.5	37.5	37.5	39.1	42.5
22/01/1987	22:00	18.5	33.3	35.1	35.5	36.1	36.3	36.3	23.0	24.7	38.0	55.9	62.5	63.1	37.8	41.3	55.0	73.5	80.3	80.9
24/10/1987	12:00	14.4	17.8	21.1	21.8	21.8	21.8	21.8	18.0	25.2	35.0	39.8	46.5	50.1	21.4	31.9	42.4	47.2	53.9	57.5
26/09/1989	00:00	29.4	29.4	29.4	29.4	29.4	29.4	29.4	47.0	70.1	88.3	92.6	92.6	96.5	47.0	70.1	88.3	92.6	92.6	96.5
18/09/1990	14:00	37.7	37.7	37.7	37.7	37.7	37.8	37.8	37.8	37.8	37.8	37.8	37.8	37.8	37.8	37.8	37.8	37.8	37.9	37.9
08/12/1990	01:00	31.0	40.2	40.2	40.2	40.2	40.2	40.2	32.0	33.9	33.9	35.0	36.6	36.6	41.2	43.1	43.1	44.2	45.8	45.8
29/10/1991	11:00	25.4	41.8	41.8	41.8	49.0	49.0	49.0	28.1	28.1	28.1	28.1	28.1	28.1	44.5	44.5	44.5	51.7	51.7	51.7
09/05/1993	10:00	18.6	19.8	20.2	35.0	48.0	54.3	55.6	18.7	21.6	21.6	24.6	24.6	24.6	19.9	23.2	38.0	54.0	60.3	61.6
29/10/1993	03:00	29.8	33.4	40.0	47.7	57.1	62.3	64.4	29.8	30.8	31.0	31.1	31.3	31.7	33.4	41.0	48.9	58.4	63.8	66.3
07/10/1994	16:00	10.9	14.5	14.8	14.8	14.8	14.9	14.9	20.0	29.2	36.4	41.4	44.1	44.8	23.6	33.1	40.3	45.3	48.1	48.8
22/03/1996	13:00	32.5	37.8	37.9	38.0	38.4	38.4	38.4	32.5	32.5	32.5	32.5	32.5	32.5	37.8	37.9	38.0	38.4	38.4	38.4
19/03/1997	05:00	15.9	15.9	15.9	15.9	15.9	15.9	15.9	18.2	24.9	25.5	25.5	25.5	26.0	18.2	24.9	25.5	25.5	25.5	26.0
01/02/1998	03:00	28.7	32.1	33.0	35.4	37.6	37.6	37.8	46.7	49.8	50.1	50.1	50.2	51.8	50.1	54.1	56.8	59.0	59.1	60.9
05/11/1998	21:00	11.9	19.2	20.1	20.1	20.1	20.1	20.1	21.3	21.4	21.4	21.4	21.4	21.4	28.6	29.6	29.6	29.6	29.6	29.6
10/10/1999	03:00	26.5	39.8	39.8	39.8	39.8	39.8	39.8	26.5	26.5	26.5	26.7	26.7	26.7	39.8	39.8	39.8	40.0	40.0	40.0
18/12/2000	23:00	20.4	21.6	21.7	21.7	21.7	21.7	21.7	28.5	30.0	34.8	35.8	35.9	36.1	29.7	31.3	36.1	37.1	37.2	37.4
18/11/2001	13:00	20.6	35.8	51.8	56.0	58.6	58.8	58.8	21.6	22.5	22.6	22.7	22.7	22.7	36.8	53.7	58.0	60.7	60.9	60.9
24/11/2002	14:00	29.9	44.1	50.6	51.9	53.0	53.8	53.9	45.4	52.9	54.1	54.1	54.1	54.1	59.6	73.6	76.1	77.2	78.0	78.1
10/10/2003	23:00	18.4	21.8	21.8	21.8	25.1	25.1	25.1	25.0	25.1	25.1	25.1	25.1	25.1	28.4	28.5	28.5	31.8	31.8	31.8
17/10/2004	00:00	21.4	31.2	34.4	35.3	36.2	40.5	45.1	21.5	21.7	21.7	21.7	23.0	29.2	31.3	34.7	35.6	36.5	42.1	52.9
24/01/2006	10:00	15.5	16.7	16.7	16.7	16.7	16.7	16.7	16.9	17.5	17.8	22.8	22.8	22.8	18.1	18.7	19.0	24.0	24.0	24.0
07/04/2007	22:00	22.1	22.5	22.5	22.5	23.1	32.5	38.3	40.8	40.8	40.8	40.9	40.9	40.9	41.2	41.2	41.2	41.9	51.3	57.1
08/04/2008	09:00	41.4	45.6	46.1	46.9	47.0	47.6	55.8	42.9	42.9	42.9	42.9	43.4	66.1	47.1	47.6	48.4	48.5	49.6	80.5
26/12/2008	22:00	17.2	22.5	22.7	24.2	24.9	24.9	24.9	20.7	20.7	21.3	24.0	31.1	34.2	26.0	26.2	28.3	31.7	38.8	41.9
20/02/2010	10:00	51.2	80.2	91.0	96.3	98.8	101.5	102.8	62.3	71.6	73.8	75.1	78.0	89.5	91.3	111.4	118.9	122.7	128.3	141.1
25/11/2010	14:00	37.3	46.7	47.6	47.6	47.6	47.6	47.6	39.0	44.2	46.9	58.1	61.4	81.6	48.4	54.5	57.2	68.4	71.7	91.9
23/10/2011	23:00	10.2	13.8	14.1	14.1	14.1	14.3	14.4	18.5	23.3	23.7	23.7	24.6	24.7	22.1	27.2	27.6	27.6	28.7	28.9
25/11/2012	01:00	21.5	27.9	38.1	45.7	50.9	55.5	57.3	21.6	24.4	30.3	30.4	30.6	30.8	28.0	41.0	54.5	59.8	64.6	66.6
18/10/2013	14:00	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7	21.7

After all the marginal models were fitted to the series, tested and the best selected, each series of rainfall or cumulative rainfall must have its values made dimensionless, by transforming the original values into 0 to 1 values, according to the marginal distribution that best fitted the series. The dimensionless series are the ones to be considered in the establishment of the copulas.

Once all the dimensionless series were calculated, the copulas can be modelled. For this purpose, different copula types were also compared, tested and selected. The selected copulas were then studied for nonlinear correlations and return periods. The following 22 copulas were tested (identification according to the "VineCopula" R package, https://cran.r-project.org/web/packages/VineCopula/): Independence copula; Gaussian copula; Student-t copula; Clayton copula; Gumbel copula; Frank copula; Joe copula; BB1 copula; BB6 copula; BB7 copula; BB8 copula; rotated Clayton copula; rotated Gumbel copula; rotated Joe copula; rotated BB1 copula; rotated BB6 copula; rotated BB7 copula; rotated BB8 copula; Tawn type 1 copula; rotated Tawn type 1 copula; Tawn type 2 copula; rotated Tawn type 2 copula. Each rotation was 180º.

The relative fitting quality of the different copulas was assessed based on the three estimators LLF, AIC, and BIC in much the same way as for the analyses of the marginal distributions.

4. RESULTS

The best 18-fitting copula families adopted for further study are identified in Table 3, along with the values of their parameters and of Kendall's coefficient, τ, among coupled rainfall series.

Based on the copula cumulative distribution functions previously established, joint and conditional return periods were computed. Figure 3 exemplifies some of the results achieved based on the representation of the contour lines of the "or" and "and" joint return periods as a function of the coupled (X_0, X_n^{BA}) extreme rainfall events and by highlighting from those events those that were identified as having originated alluvium floods. Such type of representation is the more suitable one since various combinations of the coupled events can have the same return period.

Table 3. Copula families and parameters and Kendall's rank correlation coefficient among coupled variables.

	n	Copula family	Parameter 1	Parameter 2	τ
Rainfall before	1	Gaussian	0.91	-	0.73
	2	Gaussian	0.87	-	0.67
	3	BB7 copula	1.86	2.45	0.6
	4	BB7 copula	1.65	2.38	0.58
	5	BB7 copula	1.56	2.39	0.58
	6	BB7 copula	1.65	2.34	0.58
Rainfall after	1	Gaussian	0.86	-	0.66
	2	Frank copula	7.22	-	0.57
	3	Gaussian	0.68	-	0.47
	4	Rotated Twan type 1 copula	2.45	0.62	0.42
	5	Rotated Twan type 1 copula	2.38	0.59	0.39
	6	Gaussian	0.63	-	0.43

	n	Copula family	Parameter 1	Parameter 2	τ
Rainfall before and after	1	Gaussian	0.82	-	0.62
	2	Gaussian	0.75	-	0.54
	3	Gaussian	0.68	-	0.48
	4	Gaussian	0.64	-	0.44
	5	Gaussian	0.61	-	0.42
	6	Gumbel copula	1.83	-	0.45

In each diagram, the x-axis refers to the hourly AMS series (X_0), and the y-axis to cumulative rainfall series of rainfall from 1 to 6 h temporarily contiguous to the annual maxima and including the maxima. From the 34 coupled rainfall events, the white circles represent the 23 that were not identified as alluvium-trigger events, the red and slightly bigger orange circles represent the 11 rainfall events of Table 1 that are associated with alluvium events, the orange one referring to the late February 2010 alluvium flooding. The contour lines for the hourly AMS rainfall separately coupled with the series of cumulative rainfalls before, (X_0, X_n^B), and after, (X_0, X_n^A), the AMS have similar general shapes.

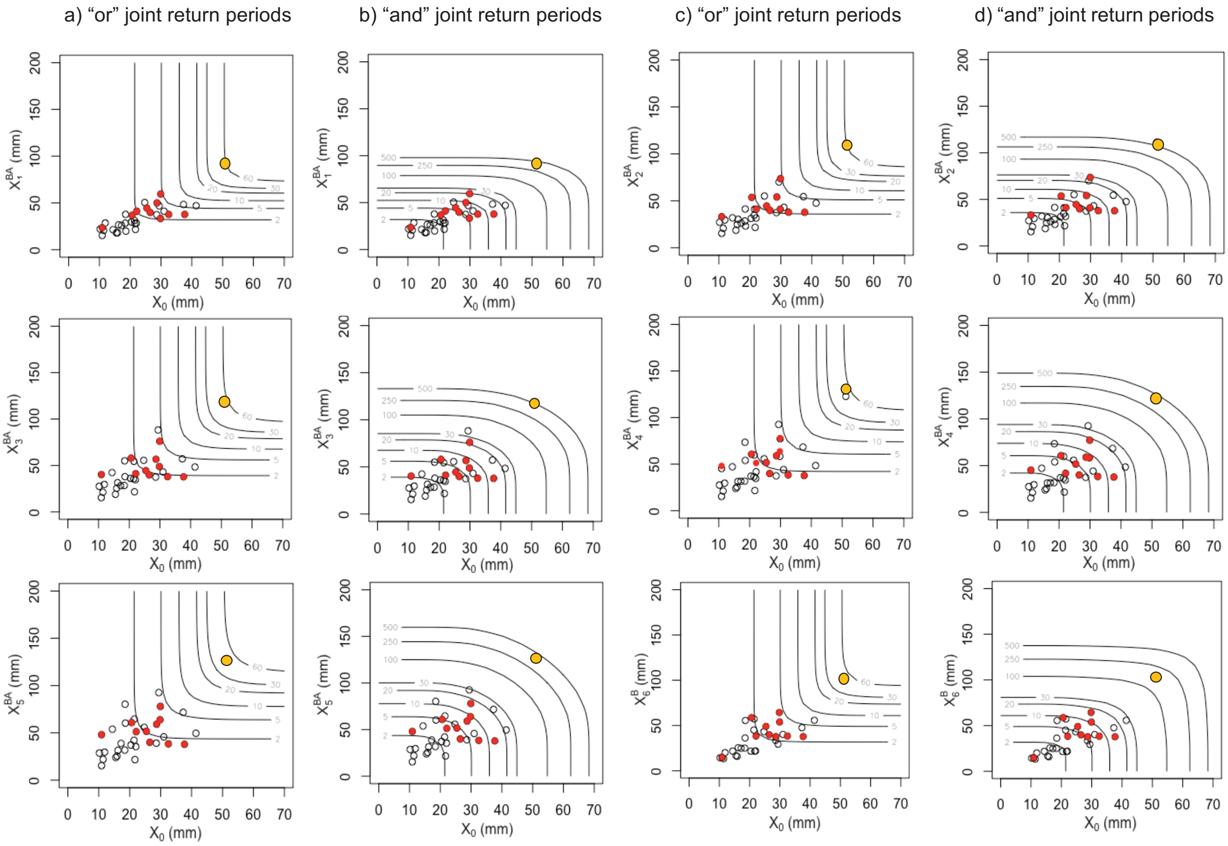

Figure 3. Contour lines of joint "or" and "and" return periods (years) for coupled hourly AMS (X_0) and cumulative hourly rainfalls in 1 to 6 h before and after the annual maximum, X_1^{BA} to X_6^{BA}. The red and the slightly bigger orange circles represent the coupled (X_0, X_n^{BA}) extreme rainfall events to which alluvium flood coupled, the orange one referring to the deadly event of late February 2010.

The figure exemplifies the differences between "or" and "and" joint return periods, the latter ones being always considerably higher because they express the probability of the events formed by each two coupled rainfalls instead of considering separately and alternatively those rainfalls. This demonstrates how relevant is to consider the association of rainfall events when characterizing the exceptionality of the extreme occurrences.

Figure 3 also shows that the 2010 late February rainfall event is always set apart from the other 33 extreme rainfall events represented, thus showing its true exceptionality, in relative terms, but also absolute terms. This is especially evident for the “and” results and values of “n” up to 5, with joint return periods close to 500 years while all the remaining rainfall events that generated alluviums have return periods of ca. 30 years.

There is also a noticeable tendency for rainfall events that were associated with alluviums to have higher return periods than those without alluviums assigned. Especially for values of n from 3 to 5, there is, however, an exception for three events that were not coupled with alluvium events which suggests that extreme rainfalls may have different consequences depending on the antecedent wetness conditions of the watersheds. The figure also shows that the coupled extreme rainfall events become less clustered as the duration of the cumulative rainfalls increases.

As part of this study, two types of conditional return periods were also calculated: the return period of AMS events given the rainfall cumulative events and the return period of the cumulative rainfall events given the AMS events.

The conditional return periods also identified the 2010 late February event as being truly exceptional. However, the nature of conditional probabilities resulted in return periods far beyond understandable values. This is especially notable for the 2010 February rainfall event with return periods over ten thousand or even over one hundred thousand years! For example, the return period of the AMS given the cumulative rainfall 2 h before and after the annual maximum is $T_{X_0|X_2^{BA}} = \frac{T_{X_2^{BA}}}{P(X_0 \geq x_0, X_2^{BA} \geq x_2^{BA})} = 208396$ years. All other conditional return periods are in general much larger than those from the joint analyses and most of the time unreasonable higher making hard to adopt them as measures of the exceptionality of the rainfall events.

5. DISCUSSION AND CONCLUSIONS

The results previously presented exemplify how the copula analysis can contribute to understanding the way the temporal distribution of rainfall during a rainfall event constrains its exceptionality, especially with the use of the “and” joint return periods.

The analysis also stressed the exceptionality of the deadly late February 2010 event. However, joint and conditional probabilities do not result in similar return periods, even in the x10 to the n^{th} power. Conditional return periods were often unreasonably high, suggesting that they may not be adequate to characterize the rainfall events. The “and” joint return periods were larger than its “or” counterparts, which is conceptually understandable. However, the “or” combination might not provide the most exact probability values when trying to relate extreme rainfalls to another event, in this case, alluvium flooding. This is because the “or” combination expresses the return period of either the annual maximum (or greater) happening or the cumulative, which also includes the maximum (or greater) happening. This research concludes that the “and” joint return period values might be the best estimate of the return periods of extreme rainfall events and their associated alluvium events.

For the late February 2010 rainfall event, Figure 4 was obtained to further compare the “or” and “and” bivariate results, between them but also with those from the univariate approach applied to the different rainfall series, including the hourly AMS. The figure also provides additional insights into the characteristics and exceptionality of such rainfall events.

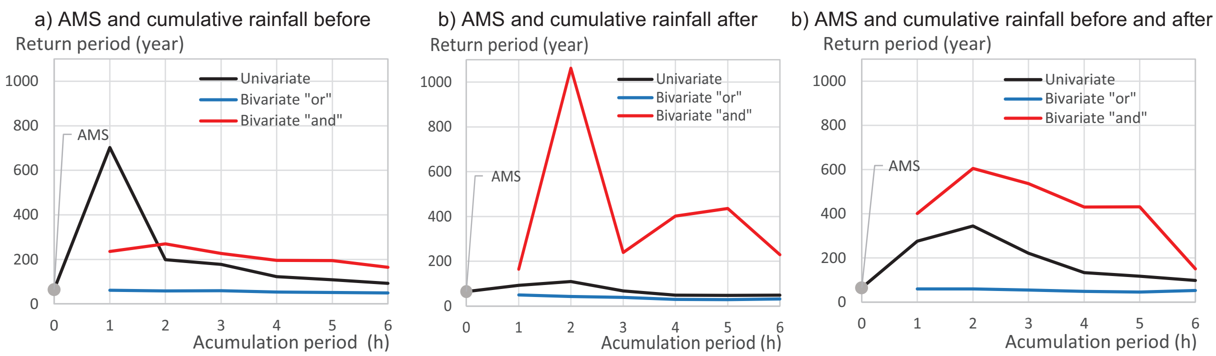

Figure 4. Rainfall event of the 20th of February 2010. Univariate and bivariate return periods for the annual maximum hourly rainfalls (AMS) and the cumulative rainfall, from 1 to 6 h: a) before; b) after; and c) before and after.

Based on the previous figure a conclusion can be made that the univariate approach also results in relatively high return periods, particularly for the cumulative rainfall 1 h before the annual maximum (ca. 700

y) and 1 and 2 h before and after this maximum (ca. 275 and 340 y, respectively). The highest return period given by the bivariate “and” approach relates to cumulative rainfalls of 2 h before the annual maximum (ca. 1060 y), which also contributed to the very high return period obtained for 2 h before and after that maximum (ca. 605 y). The differences between joint and univariate return periods and between "or" and "and" joint return periods are seemingly apparent.

In any case, the results indicate that to understand the consequences, in terms of alluviums, of a given rainfall event it is important to look at “coupled” extreme rainfalls, namely under a “and” perspective, as the best way of getting a descriptor of the wetness conditions able of triggering such type of floods.

Another conclusion can be made regarding the use of the AMS technique: despite its simplicity in terms of models and data requirements, it does not capture the fullness of the extreme rainfall data because it discards values that, despite being higher than some of the AMS series, are smaller than the maxima in the years they relate to. A possible improvement could be to use of an alternative sampling technique, namely, the Peaks Over Threshold, POT, technique (Liu *et al*., 2013, Mase, 1996). In the POT technique, any value above a prefixed threshold is considered an extreme value provided some theoretical prerequisites are met (Silva *et al.*, 2012). This would result in longer extreme rainfall series with, on average, more than one value per year, and, in turn, possibly more accurate fittings for marginal distributions and higher accuracy of the copula models.

It should be pointed out that it is important to analyse other areas of Madeira Island. Many of the alluvium events identified by Sepúlveda (2011) were not considered for coupling with extreme rainfall events because they were not reported as having affected the Funchal area (spatial criterion). A further development of the analysis of these additional areas, but also in the Funchal area, could include the implementation of copula models in a multivariate perspective, by considering other variables (e.g., related to geomorphology or terrain slopes) able of quantifying the “degree of destruction” or the “intensity” of the alluvium flood events that were coupled with extreme rainfall events. This would be a possible improvement over the current binary system of coupled rainfall-alluvium events, because it would allow to understand if a more exceptional rainfall produced more exceptional alluvium flooding events.

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